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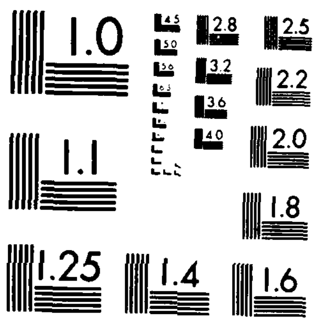

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A CURVE-FITTING METHOD FOR THE ANALYSIS OF DATA OBEYING AN ASSUMED ANALYTIC RELATIONSHIP

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J. S. Smith

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SUMMARY

A simple curve-fitting method is described, having application to data which obeys an assumed analytic relationship, but in which an unknown error or offset is present in the independent variable. The use of the method is illustrated by showing how it may be used in the calibration of accelerometers and the analysis of drag measurements.

LIST OF CONTENTS

		Page
1	INTRODUCTION	3
2	CURVE-FITTING METHOD	3
3	EXAMPLES OF APPLICATIONS OF THE METHOD	5
	3.1 Accelerometer calibration	5
	3.2 Lift-dependent drag analysis	7
4	CONCLUSIONS	10
References		11
Illustrations		Figures 1-6
Report documentation page		inside back cover

1 INTRODUCTION

Curve-fitting procedures are widely used in the analysis of experimental data. The range of techniques employed is varied and includes linear regression using least-squares analysis[1], cubic-spline fitting[2] and various polynomial-fitting methods[3]. Many computer programs have been written employing the various curve-fitting methods available, and some of these have been developed to a considerable degree of sophistication[4].

Some methods, such as cubic-spline fitting[2] and the method of Ref 4, calculate a curve which passes through all the co-ordinates supplied; other methods calculate the curve of a specified form which represents the 'best' approximation to the data[1,3]. The procedure described in this Memorandum falls into the latter class of curve-fitting methods.

The method described in this Memorandum is applicable in situations where a known analytic relationship between the dependent and the independent variable may be assumed, but there exists some unknown offset error in the measurement of the independent variable. Expressed in these terms the method appears to be rather limited in scope; however, useful applications do exist and two examples are discussed in this Memorandum.

The method of curve fitting used is discussed in section 2 of this Memorandum, and examples of applications of computer programs using this method are discussed in section 3.

2 CURVE-FITTING METHOD

The method is used to fit curves to experimental data for which an analytic relationship between the dependent and the independent variables may be assumed, but in which there exists an unknown offset in the independent variable. That is curves having the form

$$y = a\,f(x - \varepsilon) + b \quad ,$$

where ε is the unknown offset error, and a and b are unknown constants. The function f is the analytic function assumed to apply to the data.

The procedure used in the curve-fitting method is to use an iterative process to find ε, using linear regression to control the iterations and to determine the coefficients a and b. The iteration for ε is performed as part of a computer program, the method adopted being outlined below.

(1) The initial data is read. This consists of the number of co-ordinates to be input and the range in which ε is expected to lie, defined by the lowest and highest values, ε_{min} and ε_{max}.

(2) The co-ordinate array of the independent variable, x, and the dependent variable, y, is stored.

(3) An array is created containing $f(x - \varepsilon_{min})$, where ε_{min} is the initial value of the unknown offset and f is the assumed analytic function.

(4) Linear regression analysis is performed on the $f(x - \varepsilon_{min})$ and y arrays, yielding values of the coefficients a and b, and the correlation coefficient of the straight line fitted to the data. The equations used are as follows[1]

$$a = \frac{\sum(XY) - \frac{\sum X \sum Y}{N}}{\sum X^2 - \frac{\left(\sum X\right)^2}{N}} ,$$

$$b = \frac{\sum Y - a\sum X}{N} ,$$

X and Y are corresponding values from the $f(x - \varepsilon_{min})$ and y arrays and N is the number of values stored in the arrays. The correlation coefficient is given by the equation

$$CC = \frac{N\sum(XY) - \sum X \sum Y}{\left[N\sum X^2 - \left(\sum X\right)^2\right]^{\frac{1}{2}}\left[N\sum Y^2 - \left(\sum Y\right)^2\right]^{\frac{1}{2}}} .$$

(5) A new array containing $f(x - \varepsilon_n)$ is created, where ε_n is a new value for ε, obtained by the equation

$$\varepsilon_n = \varepsilon_{min} + \frac{(\varepsilon_{max} - \varepsilon_{min})(n - 1)}{10} ,$$

where n is the iteration number.

(6) Linear regression is again performed and the correlation coefficient compared with that obtained in the previous iteration. If the correlation coefficient has increased the iterative process is continued. Steps 5 and 6 are repeated until a correlation coefficient is calculated which is less than that obtained in the previous iteration, or until $\varepsilon_n = \varepsilon_{max}$, in which case a warning is output indicating that the value of ε lies outside the specified range.

If the correlation coefficient is less than that calculated in the previous iteration it is presumed that the optimum value of ε lies between ε_n and ε_{n-2}. If this condition is encountered at the second iteration the warning message is output indicating that ε is outside the specified range, and the calculation stops.

(7) The difference $\varepsilon_n - \varepsilon_{n-2}$ is evaluated. If this is greater than a prescribed amount ε_{min} is set to ε_{n-2}, and ε_{max} is set to ε_n. Steps 5 to 7 are repeated until $\varepsilon_n - \varepsilon_{n-2}$ is less than the prescribed value. When this condition is satisfied ε is considered to have been established with sufficient accuracy. The values of a, b and ε are printed, together with the correlation coefficient and the root mean square error of the curve fit.

3 EXAMPLES OF APPLICATIONS OF THE METHOD

The procedure set out in the preceding section can be applied with considerable flexibility since the iterative process employed is independent of the assumed analytic function, f .

Two examples of applications of the curve-fitting method are presented in this section. The first, discussed in section 3.1, is concerned with the calibration of accelerometers, which are used to determine the model incidence in the 5m wind tunnel[5]. The second example, described in section 3.2, shows how experimental lift and drag data may be analysed to determine the lift-dependent drag parameters of a high-lift wing. This sub-section also shows some examples of the use of the derived drag parameters in data correction and analysis.

3.1 Accelerometer calibration

The 5m wind tunnel[6] is a low speed pressurised facility in which large models fitted with high-lift devices are tested. The loads generated are such that significant deflections of the models and their support rigs may occur and these deflections must be accounted for in the measurement of the model incidence. The method of measurement adopted is to use accelerometers mounted within the model as inclinometers so that a continuous measurement of the achieved model attitude may be obtained. In the 5m wind tunnel Sundstrand Q-flex servo accelerometers are used. This type of accelerometer produces an output voltage which is proportional to the acceleration component along the sensitive axis of the accelerometer.

Three accelerometers are mounted in the model in a tri-axial mount, with their sensitive axes nominally orthogonal to each other[5]. The tri-axial mount is located in a position determined by the geometry of the model; the accelerometer axes and the model axes do not necessarily coincide. For tests made with the wings level, two accelerometer outputs are significant; those from the accelerometers which, when the tri-axial mount is at zero incidence, have their sensitive axes nominally vertical, and nominally parallel with the tunnel axis.

The outputs from these two accelerometers are of the form

$$V_1 = m_1 \cos(\phi - s_\phi) \cos(\alpha - s_1) + c_1 \tag{1}$$

and

$$V_2 = m_2 \cos(\phi - s_\phi) \sin(\alpha - s_2) + c_2 \quad . \tag{2}$$

In these equations V is the output voltage, ϕ the roll angle of the model and α is the incidence of the model datum plane. m is the accelerometer sensitivity, s_ϕ the angular offset in roll, s the angular offset in pitch and c is the voltage offset (*ie* the output when $\phi = s_\phi$ and $\alpha = s$). If the tri-axial mount is truly orthogonal and the accelerometers have no axis alignment errors, s_1 and s_2 will be identical. In practice, however, the values of s calculated in the calibration of the two

accelerometers are rarely identical, the error being due partly to mounting and alignment errors and partly to the sensitivity of the curve-fitting method to scatter in the calibration data.

In calibrating the accelerometers it is assumed that

(a) tests are made with the wings of the model level. Thus $\phi = 0^{\circ}$ when the model is upright and 180° when it is inverted;

(b) the angular offset in roll, s_{ϕ} , is small.

With these assumptions the equations become

$$V_1 = m_1 \cos(\alpha - s_1) + c_1 \tag{3}$$

$$V_2 = m_2 \sin(\alpha - s_2) + c_2 \tag{4}$$

for tests with the model upright, and

$$V_1 = - m_1 \cos(\alpha - s_1) + c_1 \tag{5}$$

$$V_2 = - m_2 \sin(\alpha - s_2) + c_2 \tag{6}$$

for tests with the model inverted.

The constants m, c and s are established for each model test series using computer programs based on the curve-fitting method outlined in section 2 of this Memorandum. Calibration data for the accelerometers is obtained by recording their output voltages, V_1 and V_2 , for a number of incidences covering the anticipated experimental range, using an inclinometer as an incidence standard. The inclinometer measurements are expected to be accurate to 1 minute of arc, *ie* $\alpha = \alpha_{measured} \pm 0.017^{\circ}$.

Fig 1 shows the results of a typical static calibration of the accelerometers fitted to a model tested in the 5m wind tunnel. Fig 1a shows the calibration calculated for the accelerometer with output equation (3). The calibration coefficients obtained were $m_1 = 3.0516$, $s_1 = -2.3184^{\circ}$ and $c_1 = 0.0089$ volt. The calibration of the accelerometer with output equation (4) is shown in Fig 1b. The calculated coefficients for this accelerometer were $m_2 = 2.9322$, $s_2 = -2.3686^{\circ}$ and $c_2 = -0.0076$ volt.

The fit of the calculated calibration curves to the data was good, the correlation coefficients and root mean square error calculated for the two calibrations being

$$CC_1 = 0.9999994 \qquad \text{rms error}_1 = 0.00011 \text{ volt}$$

$$CC_2 = 0.9999988 \qquad \text{rms error}_2 = 0.00072 \text{ volt.}$$

For each accelerometer the significance of the root mean square error in volts varies with incidence. For incidences in the range $-45^{\circ} < \alpha < 45^{\circ}$ the sine-curve accelerometer (output equation (4)) is more sensitive than the cosine-curve accelerometer (output

equation (3)), and thus gives more accurate results. The root mean square errors in degrees of the two calibration curves are rms $error_1 = 0.120^\circ$, and rms $error_2 = 0.014^\circ$. Thus, despite a higher rms error in volts, the sine-curve accelerometer achieves greater accuracy than the cosine-curve accelerometer, as expected since the calibrated range was $-6^\circ < \alpha < 30^\circ$.

When a model is to be tested in both the upright and the inverted configurations the same values of m, c and s should be used for both cases, since the calibration coefficients are unaffected by inversion of the model. However, independent evaluation of the calibration coefficients will not necessarily result in an identical set being calculated for the upright and inverted cases, due to the inevitable scatter present in the calibration data.

The introduction of the roll angle, ϕ, into the accelerometer output equations allows a single set of accelerometer output constants to be derived from data obtained in separate calibrations with the model in the upright and the inverted configurations. Output equations (1) and (2) are used, with s_ϕ assumed negligible and ϕ set to 0° for the upright data and 180° for the inverted data.

Fig 2 shows the results of such a calibration for a sine-curve accelerometer (*ie* obeying output equation (2)). The calibration data was obtained for an incidence range of $-6^\circ < \alpha < 6^\circ$ for both the upright and inverted cases. The output equation calculated from the calibration data was

$$V = 2.9692 \cos\phi \sin(\alpha - 0.0086^\circ) + 0.00666 ,$$

and the rms error of the calibration curve was 0.0071°. The figure illustrates the change in gradient of the output curve associated with inversion of the model; the data having a positive gradient was obtained with the model upright ($\phi = 0^\circ$), and that with a negative gradient was obtained with the model inverted ($\phi = 180^\circ$).

3.2 Lift-dependent drag analysis

The drag polar of an uncambered, untwisted wing may be represented by the simple drag equation

$$C_D = C_{D_0} + \frac{K}{\pi A} C_L^2 ,$$

where C_{D_0} is the profile drag of the wing, which is the minimum drag and occurs at zero lift if the simple drag equation applies. The term $(K/\pi A)C_L^2$ is the drag due to lift, K being the lift-dependent drag factor, and A the aspect ratio of the wing.

This drag equation is an idealised representation of the drag polar and is applicable only to a limited range of lift coefficients. At high lift coefficients, as $C_{L_{max}}$ is approached, the occurrence of flow separations from the surface of the wing causes the drag to increase, invalidating this simple equation. When the lift coefficient is small the laminar region of the boundary layer on the wing's surface may have a significant extent, causing a reduction in the drag and again invalidating the simple drag equation.

If the wing is cambered the drag equation must be modified since, although the drag polar remains parabolic, the minimum drag no longer occurs at zero lift, but at some other lift coefficient, $C_{L_{min}}$. The modified form of the equation is

$$C_D = C_{D_{min}} + \frac{K}{\pi A}\left(C_L - C_{L_{min}}\right)^2 .$$

The applicability of this equation is affected by the same limitations as the simple drag equation and, additionally, if the cambered wing is fitted with high-lift devices, further limitations may be imposed by the occurrence of flow separations from these devices.

A computer program has been written to calculate the drag parameters $C_{D_{min}}$, K and $C_{L_{min}}$ from measured lift and drag coefficients, using the calculation method set out in section 2 of this Memorandum. The program depends on the hypothesis that the modified drag equation set out above is valid for at least a portion of the measured drag polar, and that sufficient data points have been obtained in the region of validity to allow a satisfactory fit to the data to be obtained.

The drag parameters are important because, once calculated, they determine the attached-flow drag polar, which serves as a general description of the aerodynamic performance of the aircraft or model for which it is established, in the range of validity of the drag equation used. The attached-flow drag polar also has certain specific applications, as noted below.

(a) The initial occurrence of flow separation and the changes in drag consequent on the flow separation may be established by comparison of the attached-flow and the measured drag polars. The changes in drag observed may be increments, if the separation occurs at high incidence, or decrements if the separation is from the rear surface of a slat at low incidence.

(b) In wind tunnel testing, knowledge of the separated drag is required if the blockage effects due to the separated wake are to be accounted for using the method recommended by Maskell[7]. Experience has shown that to make an objective assessment of the separated drag for a model with high-lift devices deployed the attached-flow drag polar must be represented by the modified drag equation quoted previously. The calculation method described in this Memorandum was initially developed for this application.

(c) Comparison of attached-flow drag polars may be useful in the examination of the effects of changes in high-lift device deployment angles, and the assessment of incremental effects such as those due to the carriage of stores, the lowering of the undercarriage of an aircraft, or the fitting of gauzes to restrict the flow through the engine nacelles of a wind tunnel model.

Fig 3 shows the lift and drag curves of a model of a commercial transport aircraft tested in the 5m wind tunnel in a take-off configuration, with full-span leading edge slats deployed and the flap system retracted. Drag parameters were derived from the data shown in Fig 3 and used to define the attached-flow drag polar. The drag parameters were

determined by curve-fitting the lift and drag data in the range $0.4 < (C_L/C_{L_{max}}) < 0.8$. The attached-flow drag polar is defined by the equation

$$C_D = C_{D_{min}} + \frac{K}{\pi A}\left(C_L - C_{L_{min}}\right)^2$$

where, for this set of data, $C_{D_{min}} = 0.0427$, $C_{L_{min}} = 0.2291$, $K = 1.4951$ and $A = 7.73$. The measured polar and the calculated attached-flow drag polar are shown in Fig 4.

Comparison of the attached-flow polar and the measured data shows that the initial onset of flow separation occurred at about $C_L = 1.5$. At lift coefficients of 1.62 and 1.64 increments in drag reveal the occurrence of flow separations from the two underwing engine nacelles fitted to the model, and the separated drag then increases rapidly as the lift coefficient approaches $C_{L_{max}}$.

At lift coefficients between $C_L = -0.1$ and $C_L = 0.6$ the drag of the model is less than that suggested by the attached-flow drag polar. At the low angles of incidence associated with this lift coefficient range $(-4^\circ < \alpha < +1^\circ)$, flow separation occurs from the rear surface of the slat, reducing the pressure drag which would otherwise have arisen due to the suctions on this surface. A further effect of this flow separation is a slight increase in the lift coefficient at low angles of incidence compared to that anticipated from the linear portion of the lift curve. This effect, which may be seen in Fig 3, is due to the reduced downward component of the slat load in the presence of the flow separation.

Ultimately, for angles of incidence less than -4°, the flow separation from the rear surface of the slat develops to such an extent that the increase in drag due to the separated wake outweighs the reduction in the pressure drag, resulting in the net increase in drag observed at negative lift coefficient in Fig 4.

In order to correct wind tunnel measurements for the effects of the blockage due to the wake of the model and its support rig, it is necessary to be able to derive the lift-dependent drag parameters from the measured data. The correction due to the wake blockage may be calculated using an equation of the form[7]

$$\Delta\frac{q}{q_0} = \frac{S}{2C}\left(C_{D_0} + C_{D_T}\right) + \frac{5S}{2C}\left(C_{D_{sep}}\right) .$$

In this equation $\Delta(q/q_0)$ is the correction to be applied to the wind tunnel dynamic pressure, S is the model wing area and C the tunnel cross-sectional area. C_{D_0} is the profile drag of the model and occurs at $C_L = C_{L_0}$. The profile drag is assumed to be the minimum drag in attached flow, and hence $C_{D_0} = C_{D_{min}}$ and $C_{L_0} = C_{L_{min}}$. C_{D_T} is the tare and interference drag of the model supports and $C_{D_{sep}}$ is the separated drag coefficient, given by

$$C_{D_{sep}} = C_D - C_{D_T} - C_{D_0} - C_{D_I}$$

where C_{D_I} is the lift-dependent drag

$$C_{D_I} = \frac{K}{\pi A}\left(C_L - C_{L_0}\right)^2 .$$

Fig 5 shows the variation of the measured drag with $(C_L - C_{L_0})^2$ for the data of Fig 2, and illustrates the definitions of the parameters C_{D_0}, C_{D_I} and $C_{D_{sep}}$. Also shown in Fig 5 is the calculated attached-flow drag polar, which is seen to be a very good fit to the data in the range for which the curve-fitting process was carried out, $0.2 < (C_L - C_{L_0})^2 < 1.3$. The root mean square error of the attached-flow drag polar in this range is $\Delta C_{D(rms)} = 0.0002$.

A further application of the curve-fitting program to the analysis of drag data is illustrated in Fig 6, which shows the effect of stores carriage on the drag of a model of a ground attack/training aircraft[8]. Data is presented in the Figure for four configurations:

(a) the clean configuration,

(b) with two stores pylons carried on each wing,

(c) with droptanks on the inboard pylons and the outboard pylons fitted but carrying no stores, and

(d) with droptanks on the inboard pylons, and two rocket packs fitted to a twin store carrier on each outboard pylon.

For each of these configurations the measured drag coefficient is plotted against $(C_L - C_{L_{min}})^2$ using the value of $C_{L_{min}}$ calculated for each set of data using the curve-fitting program. The lift-dependent drag parameters, defining the attached-flow drag polar for each run, are also shown in the Figure.

The increments in the profile drag coefficient and the lift-dependent drag factor resulting from the carriage of the stores are established in an objective fashion by the curve-fitting program, making the technique particularly useful for this type of comparative analysis.

4 CONCLUSIONS

The curve-fitting method described in this Memorandum forms the basis of computer programs used to calibrate accelerometers and to derive attached-flow drag polars from measured lift and drag coefficients. The results presented in this Memorandum show that the curve-fitting method works well and illustrate some of the uses of the data derived using the drag analysis computer program. Details of the various computer programs based on this curve-fitting method may be obtained through Aerodynamics Department, RAE, Farnborough.

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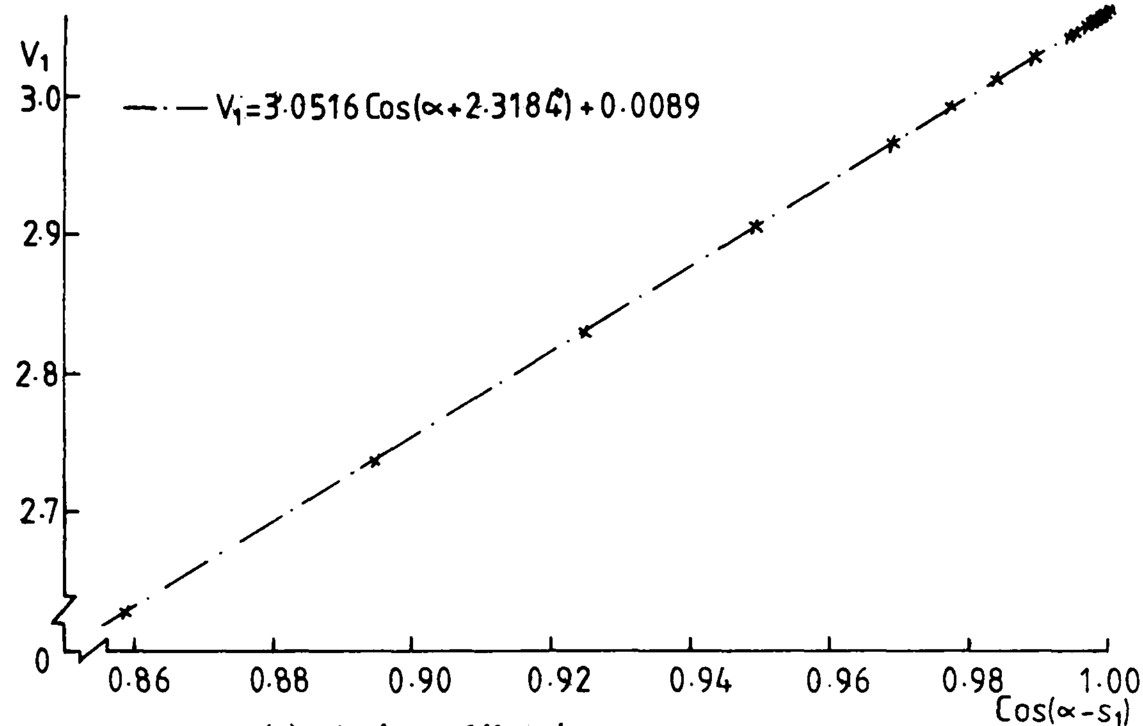
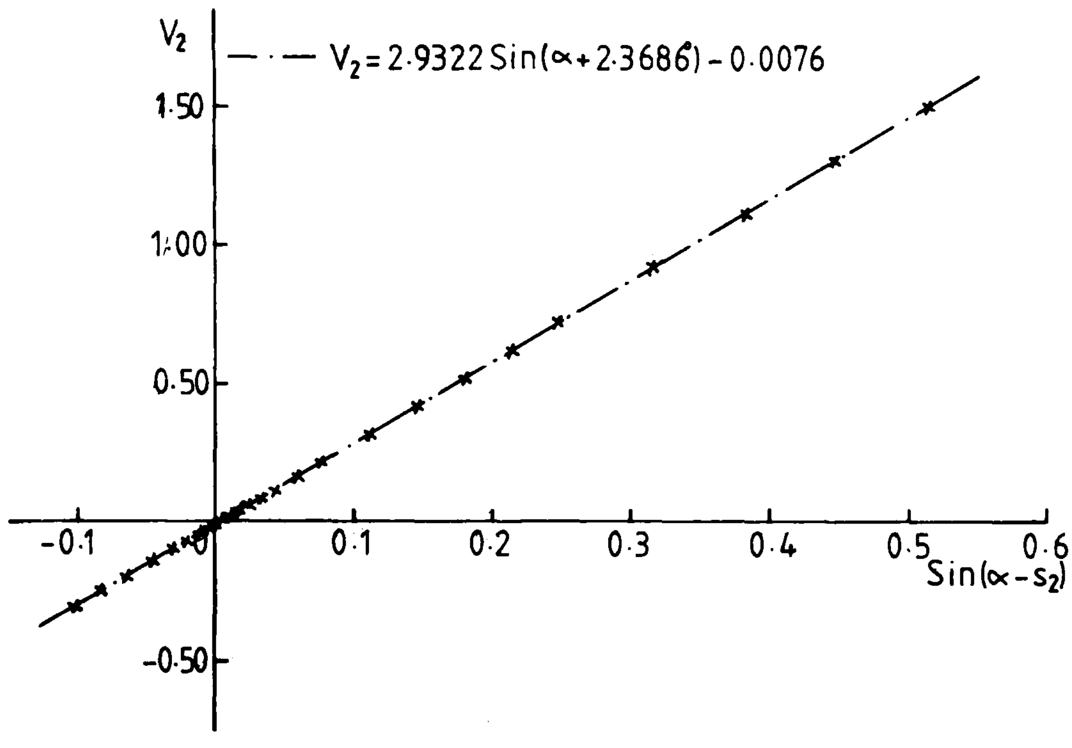


(a) Cosine calibration



(b) Sine calibration

Fig 1a&b Accelerometer calibrations

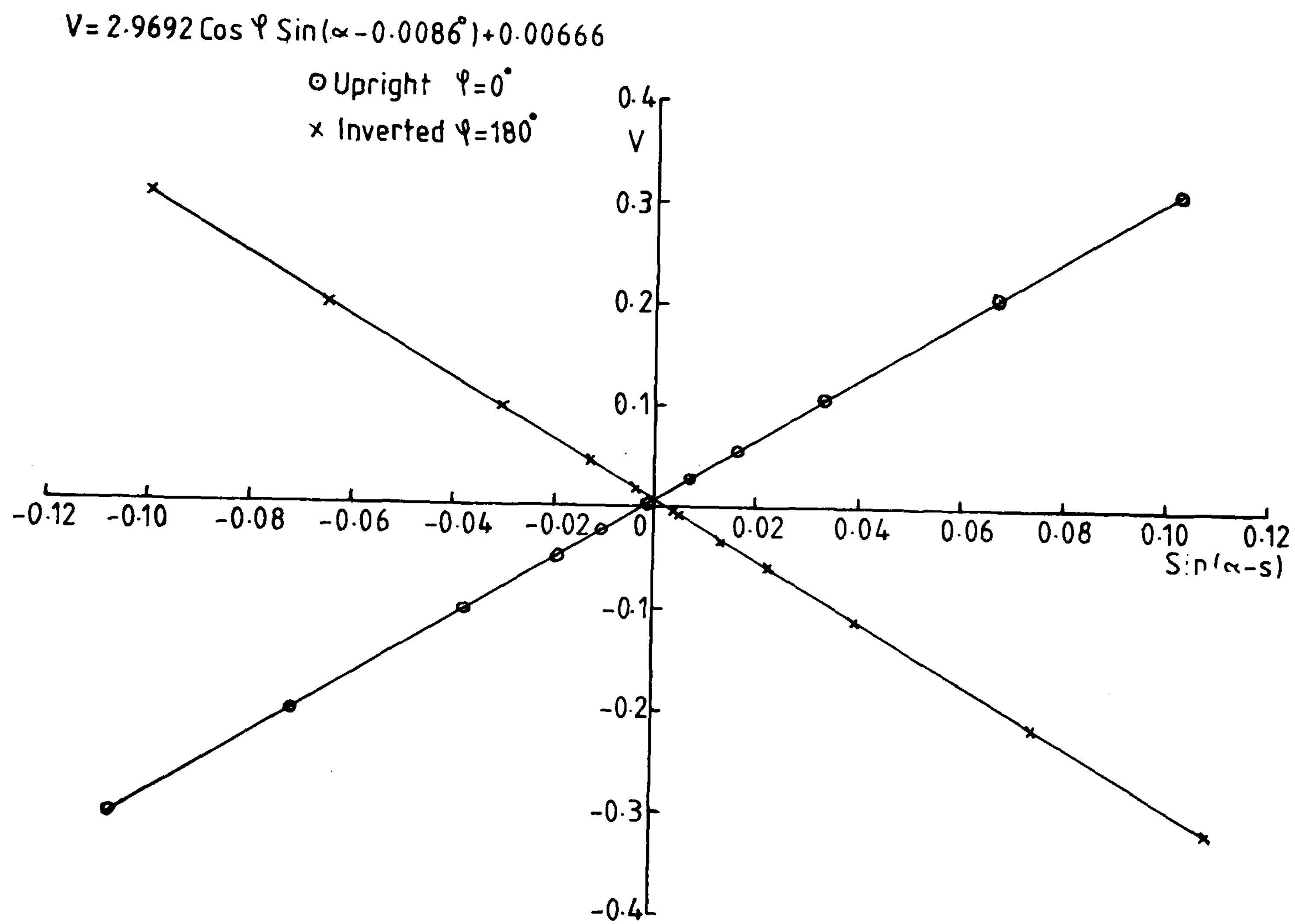


Fig 2 Accelerometer calibration for both upright and inverted data

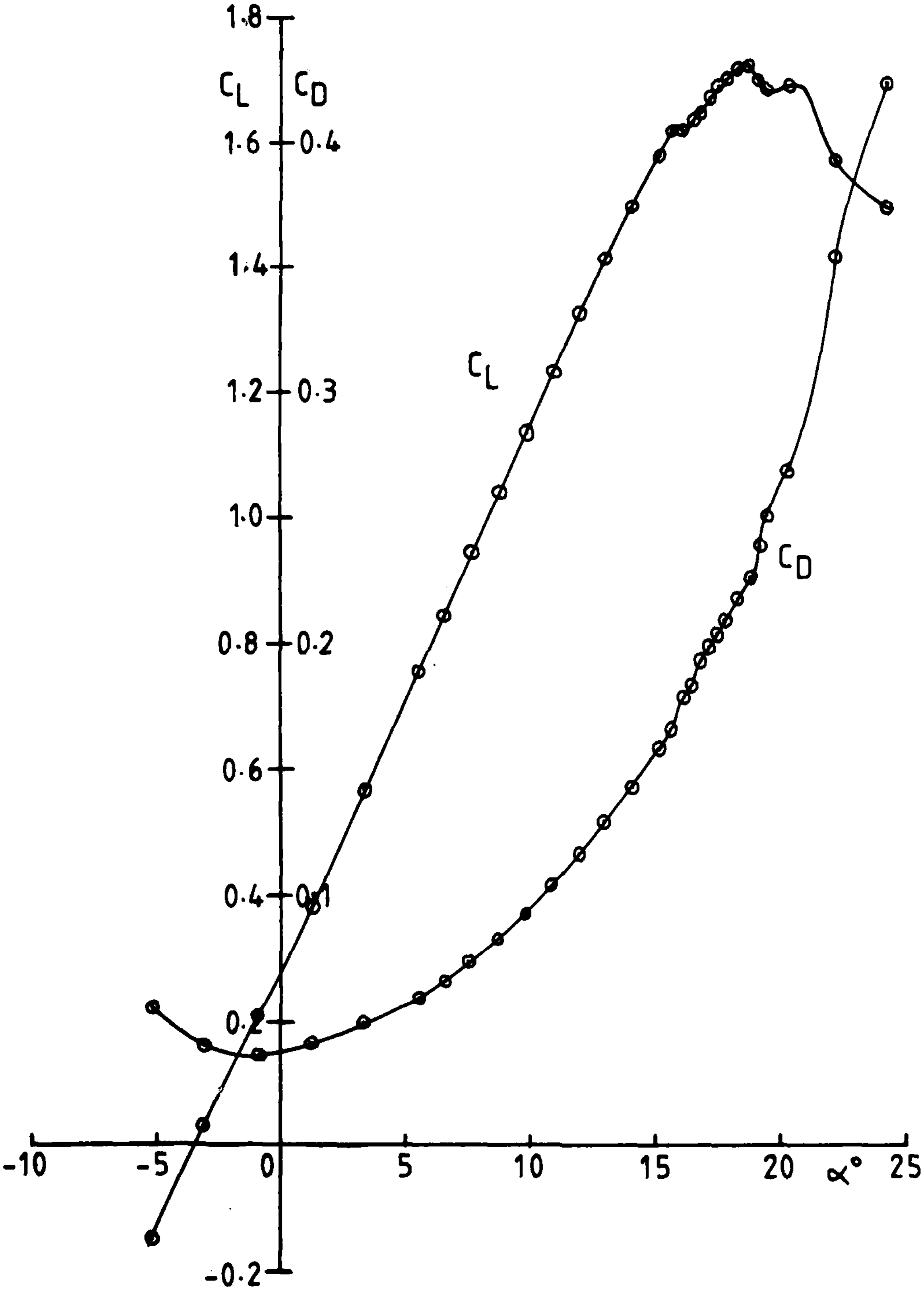


Fig 3 Lift and drag curves

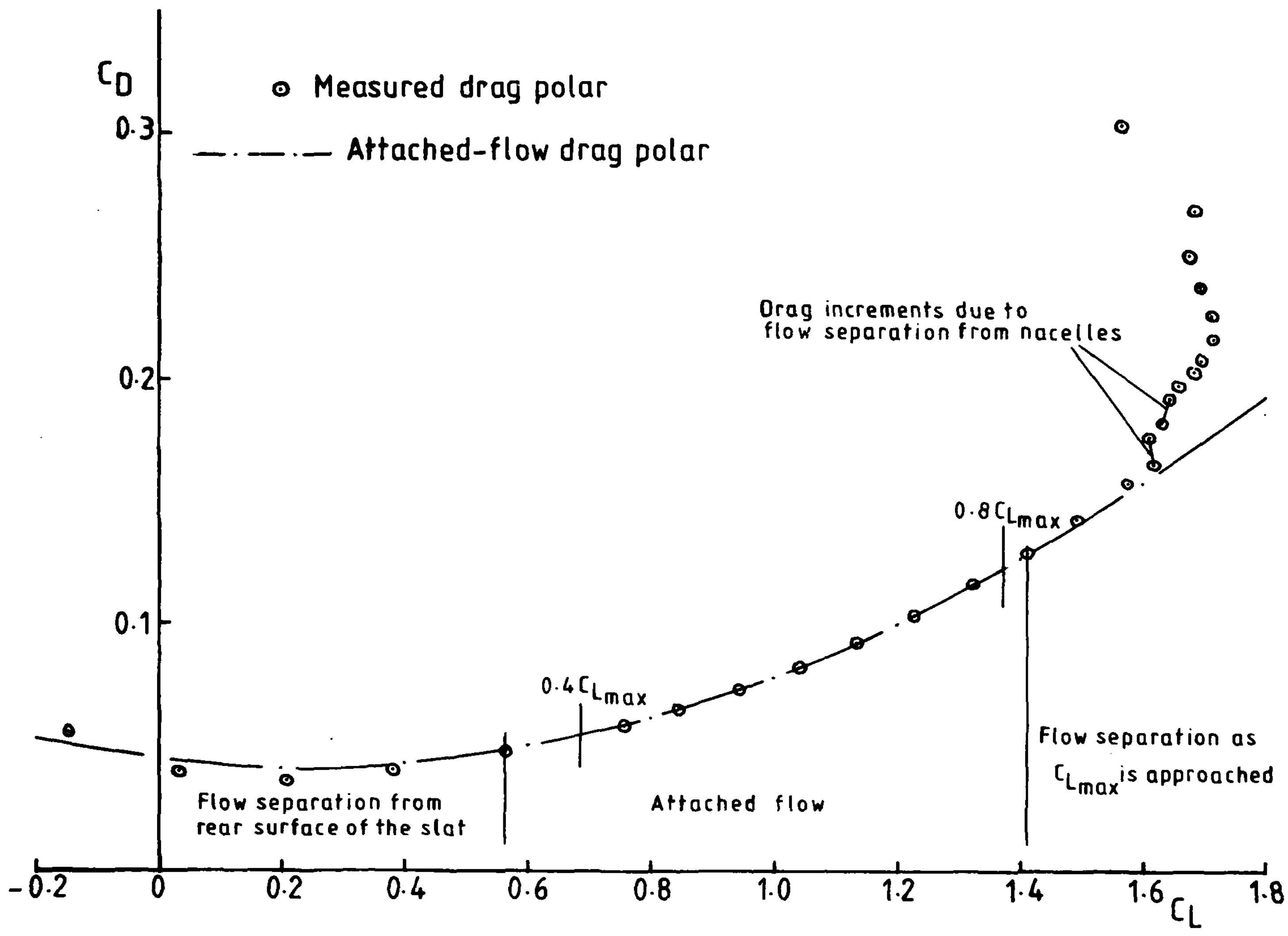


Fig 4 Measured and attached-flow drag polars

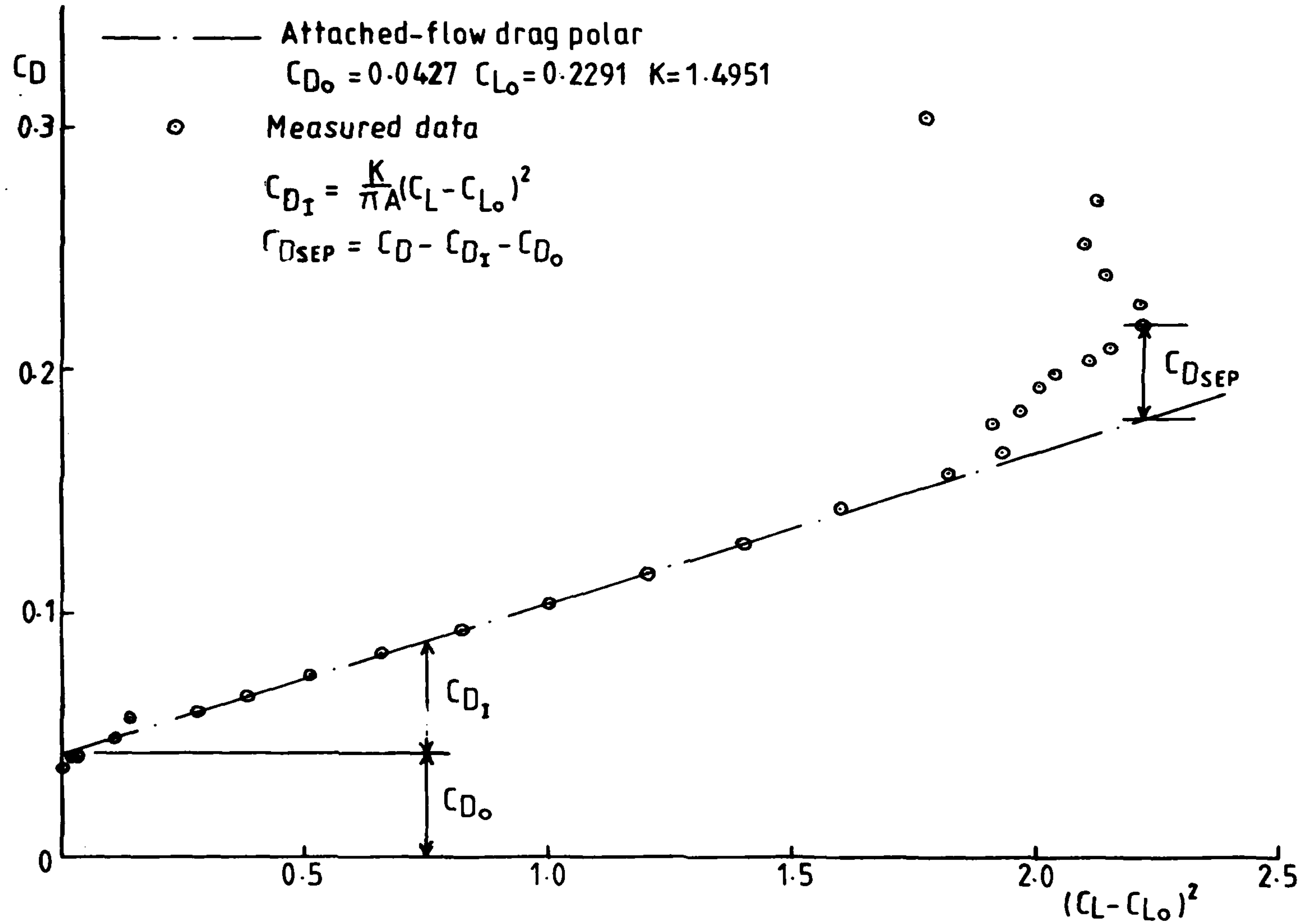


Fig 5 Parameters used in wake blockage calculation

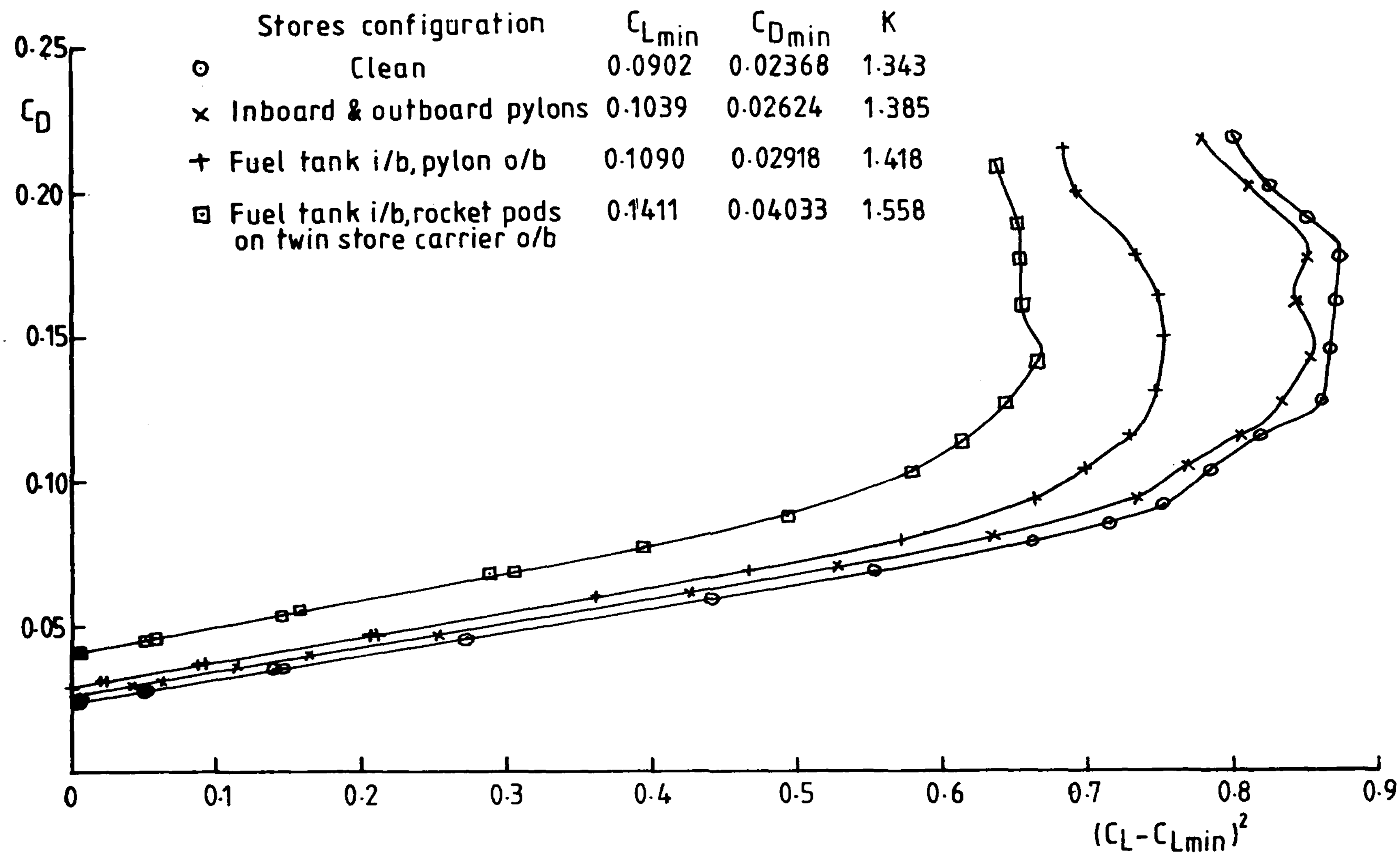


Fig 6 Effect of stores carriage

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17. Abstract

A simple curve-fitting method is described, having application to data which obeys an assumed analytic relationship, but in which an unknown error or offset is present in the independent variable. The use of the method is illustrated by showing how it may be used in the calibration of accelerometers and the analysis of drag measurements.

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